Sixteen Sorting Networks of 60 Comparators on 16 Inputs

Organised by reversal symmetry, with canonical first two layers

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on the structural account of Heath Hunnicutt

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What is a sorting network?

A **comparator network** on n wires is a fixed sequence of comparators. A comparator (i, j) with $i < j$ reads the values on wires i and j and writes back the smaller on i and the larger on j. Nothing branches: the same comparators execute in the same order whatever the input. A network is a **sorting network** if the output is sorted for every input.

Because the comparisons are fixed in advance, comparators that touch disjoint wires run simultaneously. A network therefore has two costs: its **size**, the number of comparators, and its **depth**, the number of parallel layers. The diagrams below draw wires horizontally, one per input, and comparators as vertical links; each dashed division mark separates one layer from the next, so depth is read off by counting the regions between marks.

Verification is finite. By the zero–one principle (Floyd and Knuth, 1970) a comparator network sorts all inputs if and only if it sorts all $\{0, 1\}$ vectors, so correctness at $n = 16$ is settled by checking $2^{16} = 65{,}536$ cases. Every network in this document has been checked that way, exhaustively.

What is offered here

The smallest known sorting network on 16 inputs uses **60 comparators**. That is the best known upper bound; it is not proved optimal. Exact optimal sizes $S(n)$ are established only for $n \leq 12$ (Codish, Cruz-Filipe, Frank and Schneider-Kamp, 2014–2016). Separately, **depth 9** is proved optimal at $n = 16$ (Bundala and Závodný, 2014); the networks here are of depth 10 and so trade one layer for minimum size.

The sixteen networks below all attain 60 comparators. They are offered not as an improvement on the bound — they match it — but as a **choice**. Two properties make them convenient to use:

- **Canonical opening.** All sixteen begin with the same two layers: stride one, then stride two, the Knuth/Floyd opening. Whatever else differs, the first sixteen comparators are identical and are the ones any implementation would write first.

- **Graded symmetry.** They divide by their behaviour under the reversal $w \mapsto 15 - w$, which exchanges the roles of minimum and maximum. Four are symmetric layer by layer; four more are symmetric as a whole but not within each layer; eight are not reversal-symmetric. A symmetric network is easier to lay out, easier to verify by inspection, and halves the description you have to store.

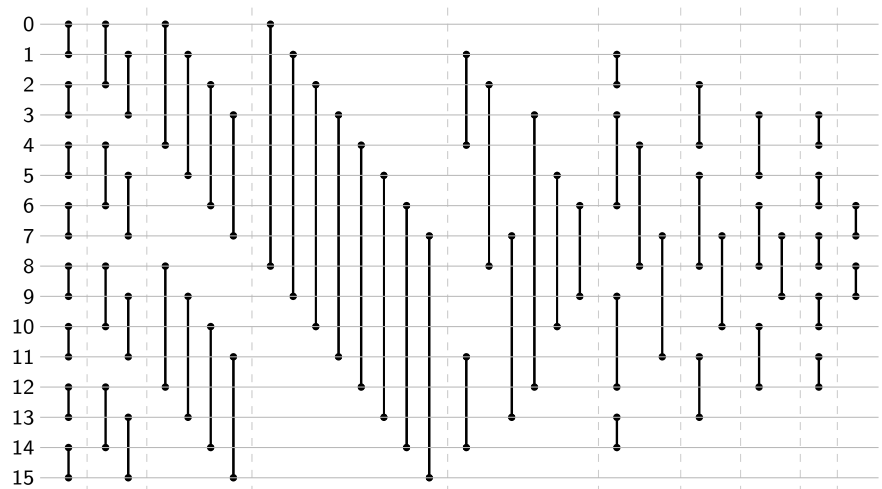
Each network is labelled by its **layer-3 program** (a, b, c), giving the relative stride taken within the minimum class $\{0, 4, 8, 12\}$, the middle class $\{1, 2, 5, 6, 9, 10, 13, 14\}$, and the maximum class $\{3, 7, 11, 15\}$ respectively. This is the first layer at which the sixteen differ.

Class I: fully symmetric

These four are invariant under $w \mapsto 15 - w$ **within every layer**. In each case the minimum class and the maximum class take the same relative stride, and the network inherits the reflection.

Network 01 $L_3 = (s_1,\ s_2,\ s_1)$

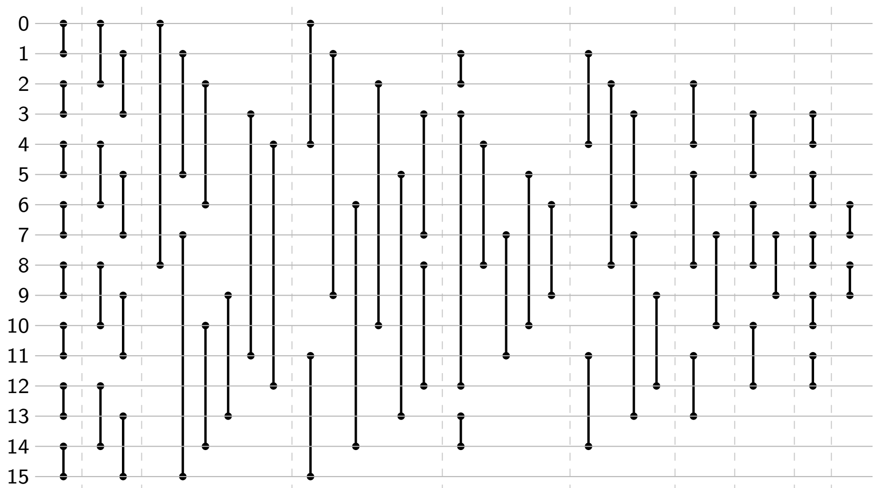
symmetric in all 10 layers.




```
L1   (0,1) (2,3) (4,5) (6,7) (8,9) (10,11) (12,13) (14,15)
L2   (0,2) (1,3) (4,6) (5,7) (8,10) (9,11) (12,14) (13,15)
L3   (0,4) (1,5) (2,6) (3,7) (8,12) (9,13) (10,14) (11,15)
L4   (0,8) (1,9) (2,10) (3,11) (4,12) (5,13) (6,14) (7,15)
L5   (1,4) (2,8) (3,12) (5,10) (6,9) (7,13) (11,14)
L6   (1,2) (3,6) (4,8) (7,11) (9,12) (13,14)
L7   (2,4) (5,8) (7,10) (11,13)
L8   (3,5) (6,8) (7,9) (10,12)
L9   (3,4) (5,6) (7,8) (9,10) (11,12)
L10  (6,7) (8,9)
```

Network 07 $\quad L_3 = (s_2,\ s_2,\ s_2)$

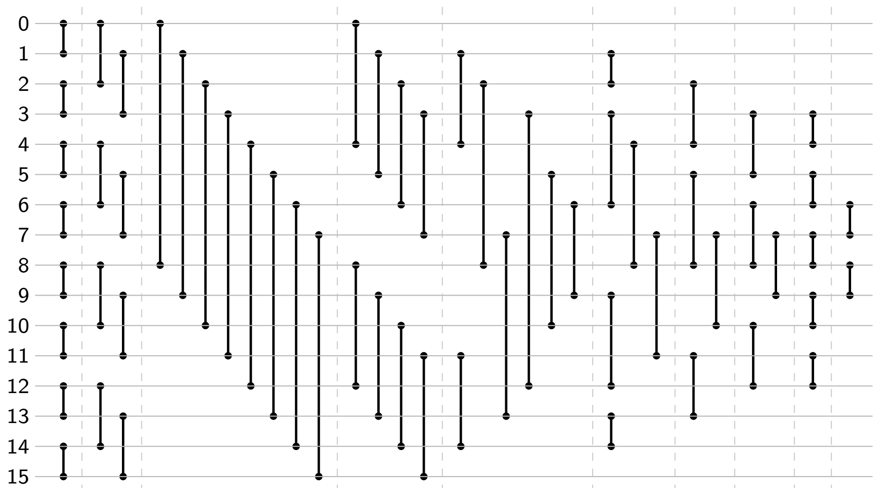
symmetric in all 10 layers.



```
L1   (0,1) (2,3) (4,5) (6,7) (8,9) (10,11) (12,13) (14,15)
L2   (0,2) (1,3) (4,6) (5,7) (8,10) (9,11) (12,14) (13,15)
L3   (0,8) (1,5) (2,6) (3,11) (4,12) (7,15) (9,13) (10,14)
L4   (0,4) (1,9) (2,10) (3,7) (5,13) (6,14) (8,12) (11,15)
L5   (1,2) (3,12) (4,8) (5,10) (6,9) (7,11) (13,14)
L6   (1,4) (2,8) (3,6) (7,13) (9,12) (11,14)
L7   (2,4) (5,8) (7,10) (11,13)
L8   (3,5) (6,8) (7,9) (10,12)
L9   (3,4) (5,6) (7,8) (9,10) (11,12)
L10  (6,7) (8,9)
```

Network 10 $\quad L_3 = (s_2,\ s_4,\ s_2)$

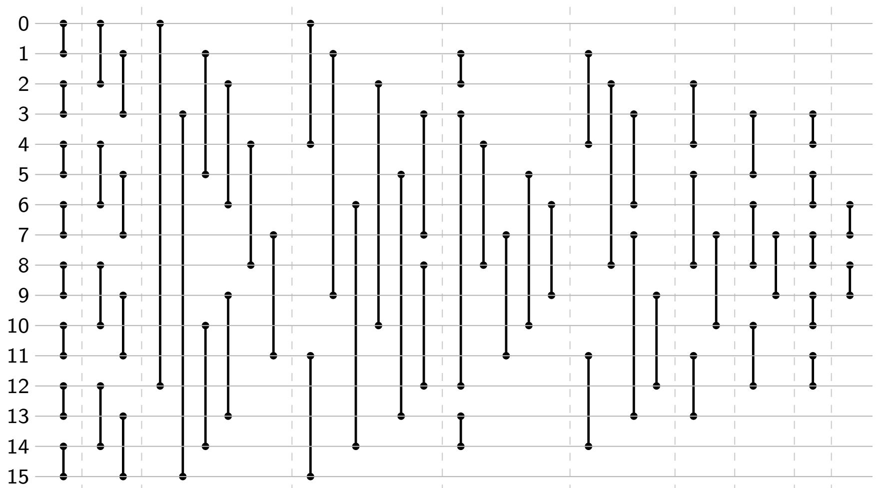
symmetric in all 10 layers.




```
L1    (0,1) (2,3) (4,5) (6,7) (8,9) (10,11) (12,13) (14,15)
L2    (0,2) (1,3) (4,6) (5,7) (8,10) (9,11) (12,14) (13,15)
L3    (0,8) (1,9) (2,10) (3,11) (4,12) (5,13) (6,14) (7,15)
L4    (0,4) (1,5) (2,6) (3,7) (8,12) (9,13) (10,14) (11,15)
L5    (1,4) (2,8) (3,12) (5,10) (6,9) (7,13) (11,14)
L6    (1,2) (3,6) (4,8) (7,11) (9,12) (13,14)
L7    (2,4) (5,8) (7,10) (11,13)
L8    (3,5) (6,8) (7,9) (10,12)
L9    (3,4) (5,6) (7,8) (9,10) (11,12)
L10   (6,7) (8,9)
```

Network 14 $L_3 = (\text{fold},\ s_2,\ \text{fold})$

symmetric in all 10 layers.



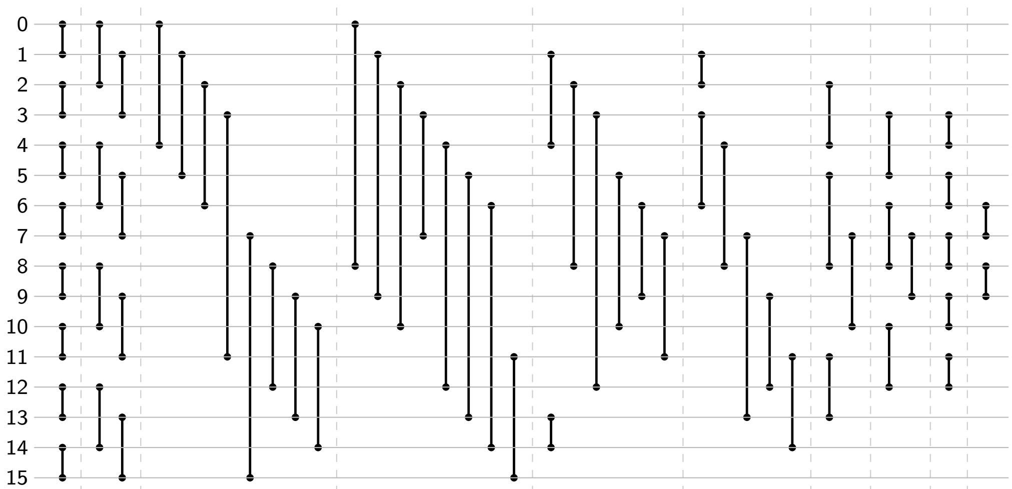

```
L1   (0,1) (2,3) (4,5) (6,7) (8,9) (10,11) (12,13) (14,15)
L2   (0,2) (1,3) (4,6) (5,7) (8,10) (9,11) (12,14) (13,15)
L3   (0,12) (1,5) (2,6) (3,15) (4,8) (7,11) (9,13) (10,14)
L4   (0,4) (1,9) (2,10) (3,7) (5,13) (6,14) (8,12) (11,15)
L5   (1,2) (3,12) (4,8) (5,10) (6,9) (7,11) (13,14)
L6   (1,4) (2,8) (3,6) (7,13) (9,12) (11,14)
L7   (2,4) (5,8) (7,10) (11,13)
L8   (3,5) (6,8) (7,9) (10,12)
L9   (3,4) (5,6) (7,8) (9,10) (11,12)
L10  (6,7) (8,9)
```

Class II: symmetric as a set

These four are carried to themselves by the reversal, but the reversal permutes layers rather than fixing each one. The comparator multiset is symmetric; the schedule is not.

Network 02 $L_3 = (s_1,\ s_2,\ s_2)$

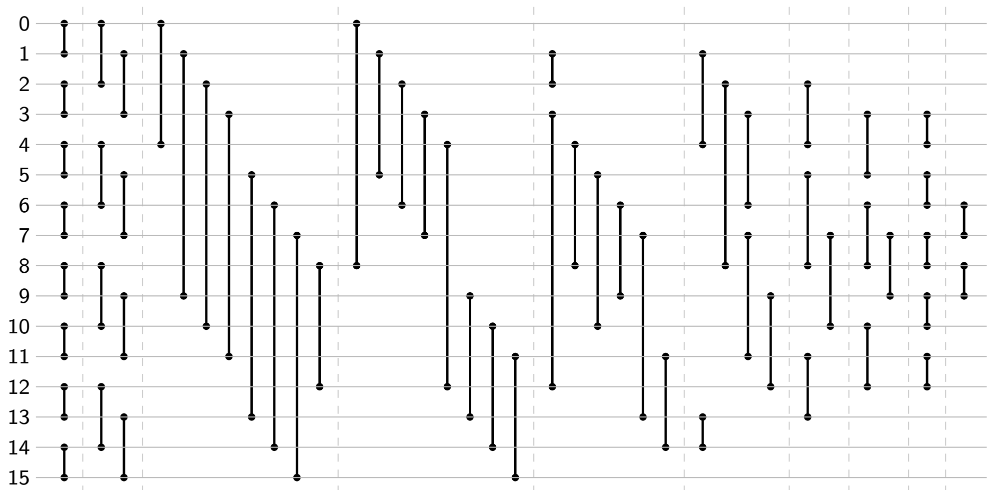
symmetric as a set; layers 1, 2, 7, 8, 9, 10 individually symmetric.




```
L1   (0,1) (2,3) (4,5) (6,7) (8,9) (10,11) (12,13) (14,15)
L2   (0,2) (1,3) (4,6) (5,7) (8,10) (9,11) (12,14) (13,15)
L3   (0,4) (1,5) (2,6) (3,11) (7,15) (8,12) (9,13) (10,14)
L4   (0,8) (1,9) (2,10) (3,7) (4,12) (5,13) (6,14) (11,15)
L5   (1,4) (2,8) (3,12) (5,10) (6,9) (7,11) (13,14)
L6   (1,2) (3,6) (4,8) (7,13) (9,12) (11,14)
L7   (2,4) (5,8) (7,10) (11,13)
L8   (3,5) (6,8) (7,9) (10,12)
L9   (3,4) (5,6) (7,8) (9,10) (11,12)
L10  (6,7) (8,9)
```

Network 04 $\quad L_3 = (s_1,\ s_4,\ s_2)$

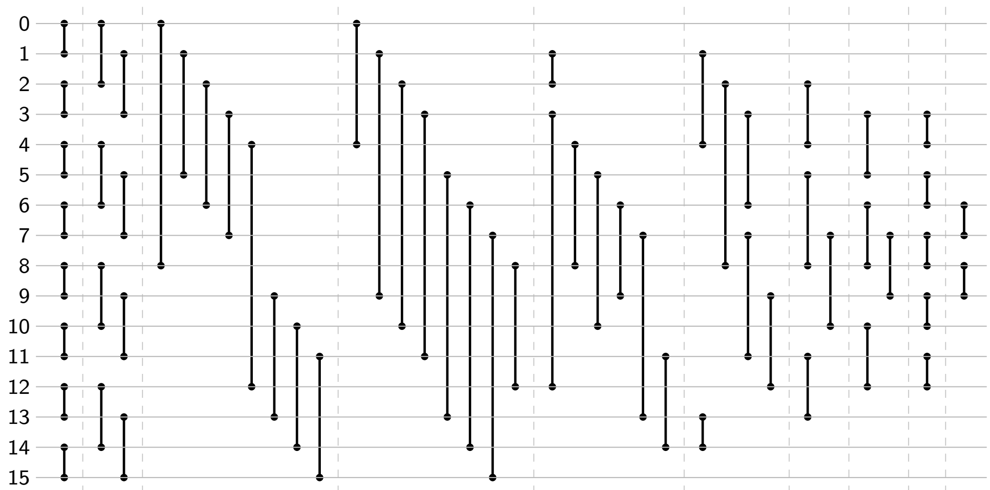
symmetric as a set; layers 1, 2, 7, 8, 9, 10 individually symmetric.




```
L1    (0,1) (2,3) (4,5) (6,7) (8,9) (10,11) (12,13) (14,15)
L2    (0,2) (1,3) (4,6) (5,7) (8,10) (9,11) (12,14) (13,15)
L3    (0,4) (1,9) (2,10) (3,11) (5,13) (6,14) (7,15) (8,12)
L4    (0,8) (1,5) (2,6) (3,7) (4,12) (9,13) (10,14) (11,15)
L5    (1,2) (3,12) (4,8) (5,10) (6,9) (7,13) (11,14)
L6    (1,4) (2,8) (3,6) (7,11) (9,12) (13,14)
L7    (2,4) (5,8) (7,10) (11,13)
L8    (3,5) (6,8) (7,9) (10,12)
L9    (3,4) (5,6) (7,8) (9,10) (11,12)
L10   (6,7) (8,9)
```

Network 06 $\quad L_3 = (s_2,\ s_2,\ s_1)$

symmetric as a set; layers 1, 2, 7, 8, 9, 10 individually symmetric.



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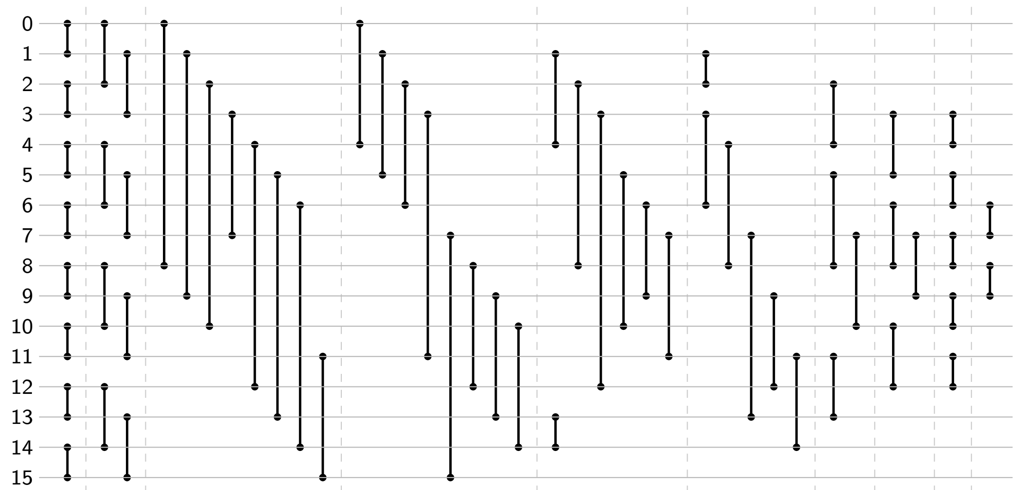
L1    (0,1) (2,3) (4,5) (6,7) (8,9) (10,11) (12,13) (14,15)
L2    (0,2) (1,3) (4,6) (5,7) (8,10) (9,11) (12,14) (13,15)
L3    (0,8) (1,5) (2,6) (3,7) (4,12) (9,13) (10,14) (11,15)
L4    (0,4) (1,9) (2,10) (3,11) (5,13) (6,14) (7,15) (8,12)
L5    (1,2) (3,12) (4,8) (5,10) (6,9) (7,13) (11,14)
L6    (1,4) (2,8) (3,6) (7,11) (9,12) (13,14)
L7    (2,4) (5,8) (7,10) (11,13)
L8    (3,5) (6,8) (7,9) (10,12)
L9    (3,4) (5,6) (7,8) (9,10) (11,12)
L10   (6,7) (8,9)
```

Network 09 $\quad L_3 = (s_2,\ s_4,\ s_1)$

symmetric as a set; layers 1, 2, 7, 8, 9, 10 individually symmetric.



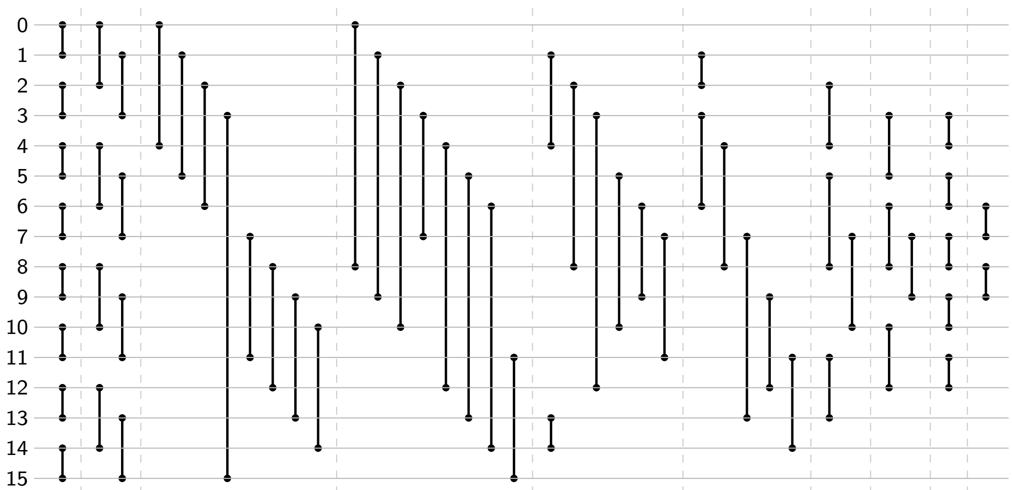

```
L1    (0,1) (2,3) (4,5) (6,7) (8,9) (10,11) (12,13) (14,15)
L2    (0,2) (1,3) (4,6) (5,7) (8,10) (9,11) (12,14) (13,15)
L3    (0,8) (1,9) (2,10) (3,7) (4,12) (5,13) (6,14) (11,15)
L4    (0,4) (1,5) (2,6) (3,11) (7,15) (8,12) (9,13) (10,14)
L5    (1,4) (2,8) (3,12) (5,10) (6,9) (7,11) (13,14)
L6    (1,2) (3,6) (4,8) (7,13) (9,12) (11,14)
L7    (2,4) (5,8) (7,10) (11,13)
L8    (3,5) (6,8) (7,9) (10,12)
L9    (3,4) (5,6) (7,8) (9,10) (11,12)
L10   (6,7) (8,9)
```

Class III: not reversal-symmetric

The remaining eight have no reversal symmetry as a whole, though several have individually symmetric layers. They are included for completeness: each is a distinct 60-comparator network with the canonical opening.

Network 03 $L_3 = (s_1,\ s_2,\ \mathrm{fold})$

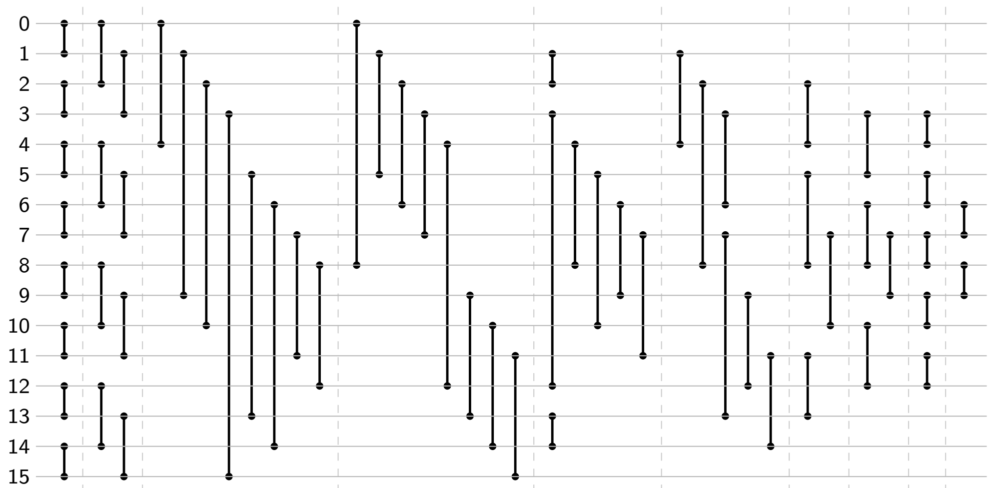
not reversal-symmetric; layers 1, 2, 7, 8, 9, 10 individually symmetric.




```
L1    (0,1) (2,3) (4,5) (6,7) (8,9) (10,11) (12,13) (14,15)
L2    (0,2) (1,3) (4,6) (5,7) (8,10) (9,11) (12,14) (13,15)
L3    (0,4) (1,5) (2,6) (3,15) (7,11) (8,12) (9,13) (10,14)
L4    (0,8) (1,9) (2,10) (3,7) (4,12) (5,13) (6,14) (11,15)
L5    (1,4) (2,8) (3,12) (5,10) (6,9) (7,11) (13,14)
L6    (1,2) (3,6) (4,8) (7,13) (9,12) (11,14)
L7    (2,4) (5,8) (7,10) (11,13)
L8    (3,5) (6,8) (7,9) (10,12)
L9    (3,4) (5,6) (7,8) (9,10) (11,12)
L10   (6,7) (8,9)
```

Network 05 $L_3 = (s_1,\ s_4,\ \mathrm{fold})$

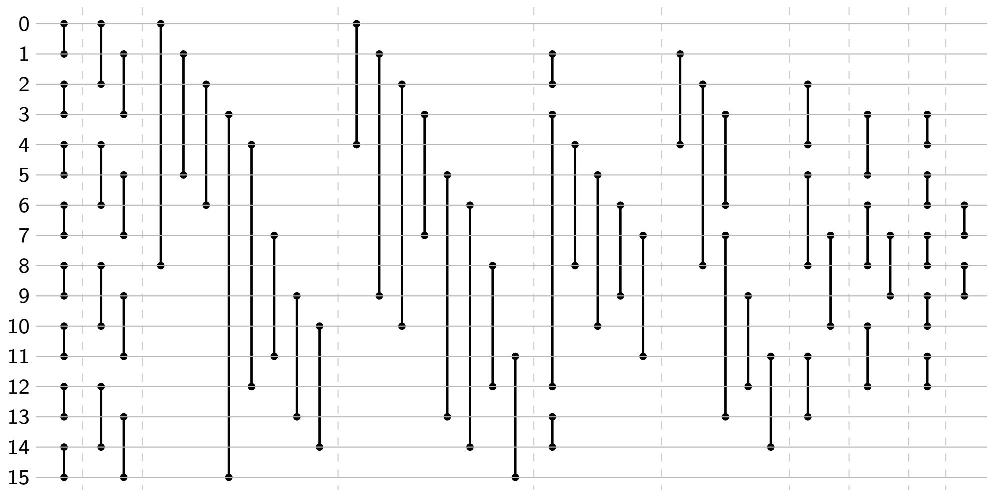
not reversal-symmetric; layers 1, 2, 5, 6, 7, 8, 9, 10 individually symmetric.




```
L1   (0,1) (2,3) (4,5) (6,7) (8,9) (10,11) (12,13) (14,15)
L2   (0,2) (1,3) (4,6) (5,7) (8,10) (9,11) (12,14) (13,15)
L3   (0,4) (1,9) (2,10) (3,15) (5,13) (6,14) (7,11) (8,12)
L4   (0,8) (1,5) (2,6) (3,7) (4,12) (9,13) (10,14) (11,15)
L5   (1,2) (3,12) (4,8) (5,10) (6,9) (7,11) (13,14)
L6   (1,4) (2,8) (3,6) (7,13) (9,12) (11,14)
L7   (2,4) (5,8) (7,10) (11,13)
L8   (3,5) (6,8) (7,9) (10,12)
L9   (3,4) (5,6) (7,8) (9,10) (11,12)
L10  (6,7) (8,9)
```

Network 08 $L_3 = (s_2,\ s_2,\ \text{fold})$

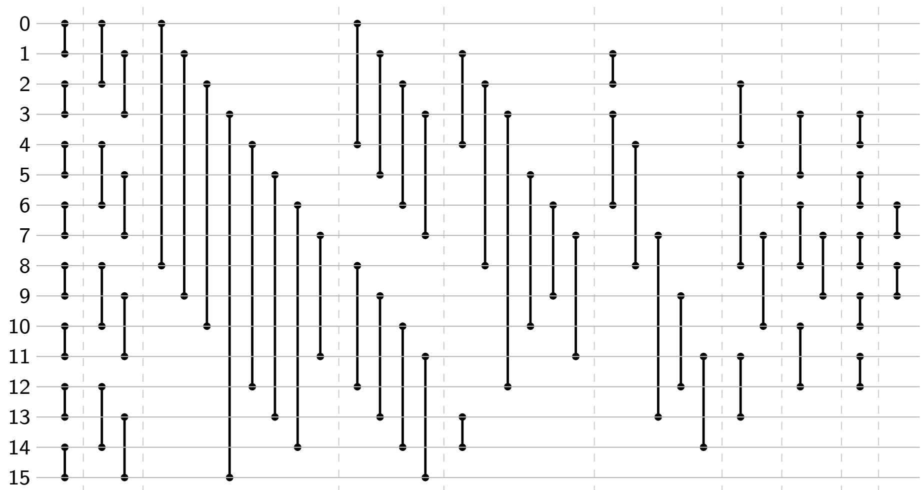
not reversal-symmetric; layers 1, 2, 4, 5, 6, 7, 8, 9, 10 individually symmetric.




```
L1    (0,1) (2,3) (4,5) (6,7) (8,9) (10,11) (12,13) (14,15)
L2    (0,2) (1,3) (4,6) (5,7) (8,10) (9,11) (12,14) (13,15)
L3    (0,8) (1,5) (2,6) (3,15) (4,12) (7,11) (9,13) (10,14)
L4    (0,4) (1,9) (2,10) (3,7) (5,13) (6,14) (8,12) (11,15)
L5    (1,2) (3,12) (4,8) (5,10) (6,9) (7,11) (13,14)
L6    (1,4) (2,8) (3,6) (7,13) (9,12) (11,14)
L7    (2,4) (5,8) (7,10) (11,13)
L8    (3,5) (6,8) (7,9) (10,12)
L9    (3,4) (5,6) (7,8) (9,10) (11,12)
L10   (6,7) (8,9)
```

Network 11 $L_3 = (s_2,\ s_4,\ \text{fold})$

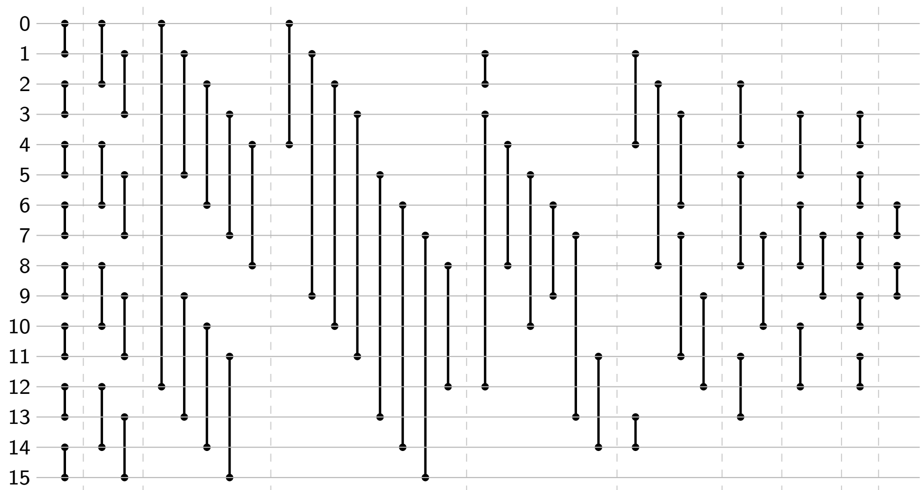
not reversal-symmetric; layers 1, 2, 4, 7, 8, 9, 10 individually symmetric.




```
L1   (0,1) (2,3) (4,5) (6,7) (8,9) (10,11) (12,13) (14,15)
L2   (0,2) (1,3) (4,6) (5,7) (8,10) (9,11) (12,14) (13,15)
L3   (0,8) (1,9) (2,10) (3,15) (4,12) (5,13) (6,14) (7,11)
L4   (0,4) (1,5) (2,6) (3,7) (8,12) (9,13) (10,14) (11,15)
L5   (1,4) (2,8) (3,12) (5,10) (6,9) (7,11) (13,14)
L6   (1,2) (3,6) (4,8) (7,13) (9,12) (11,14)
L7   (2,4) (5,8) (7,10) (11,13)
L8   (3,5) (6,8) (7,9) (10,12)
L9   (3,4) (5,6) (7,8) (9,10) (11,12)
L10  (6,7) (8,9)
```

Network 12 $L_3 = (\mathrm{fold},\ s_2,\ s_1)$

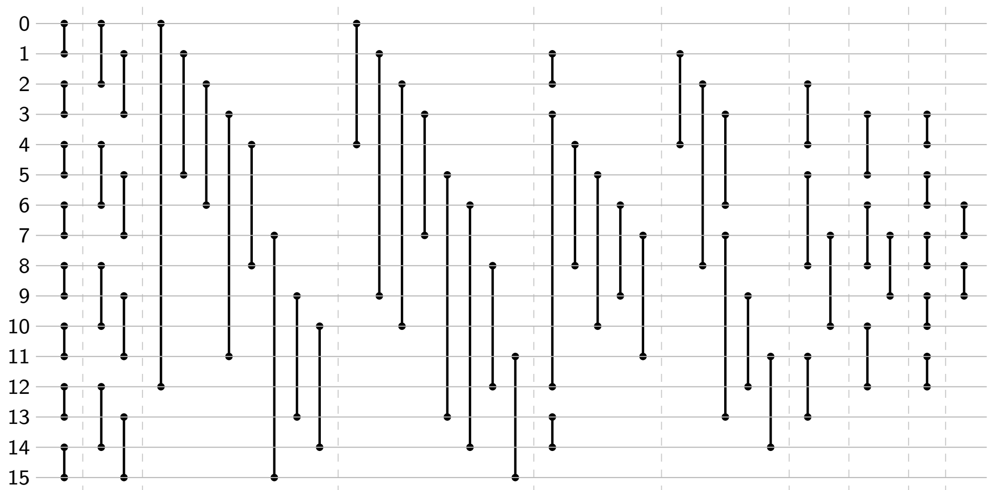
not reversal-symmetric; layers 1, 2, 7, 8, 9, 10 individually symmetric.




```
L1   (0,1) (2,3) (4,5) (6,7) (8,9) (10,11) (12,13) (14,15)
L2   (0,2) (1,3) (4,6) (5,7) (8,10) (9,11) (12,14) (13,15)
L3   (0,12) (1,5) (2,6) (3,7) (4,8) (9,13) (10,14) (11,15)
L4   (0,4) (1,9) (2,10) (3,11) (5,13) (6,14) (7,15) (8,12)
L5   (1,2) (3,12) (4,8) (5,10) (6,9) (7,13) (11,14)
L6   (1,4) (2,8) (3,6) (7,11) (9,12) (13,14)
L7   (2,4) (5,8) (7,10) (11,13)
L8   (3,5) (6,8) (7,9) (10,12)
L9   (3,4) (5,6) (7,8) (9,10) (11,12)
L10  (6,7) (8,9)
```

Network 13 $L_3 = (\text{fold},\ s_2,\ s_2)$

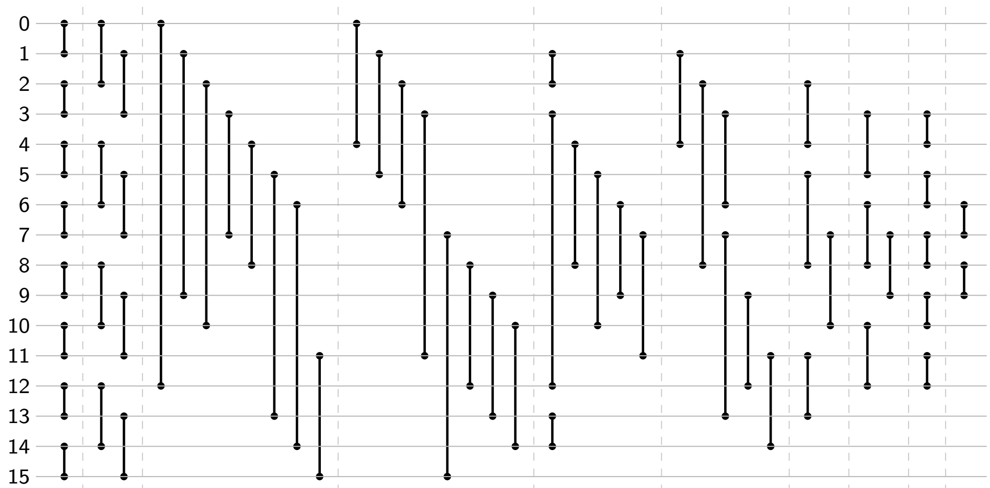
not reversal-symmetric; layers 1, 2, 4, 5, 6, 7, 8, 9, 10 individually symmetric.



```
L1    (0,1) (2,3) (4,5) (6,7) (8,9) (10,11) (12,13) (14,15)
L2    (0,2) (1,3) (4,6) (5,7) (8,10) (9,11) (12,14) (13,15)
L3    (0,12) (1,5) (2,6) (3,11) (4,8) (7,15) (9,13) (10,14)
L4    (0,4) (1,9) (2,10) (3,7) (5,13) (6,14) (8,12) (11,15)
L5    (1,2) (3,12) (4,8) (5,10) (6,9) (7,11) (13,14)
L6    (1,4) (2,8) (3,6) (7,13) (9,12) (11,14)
L7    (2,4) (5,8) (7,10) (11,13)
L8    (3,5) (6,8) (7,9) (10,12)
L9    (3,4) (5,6) (7,8) (9,10) (11,12)
L10   (6,7) (8,9)
```

Network 15 $\quad L_3 = (\text{fold},\ s_4,\ s_1)$

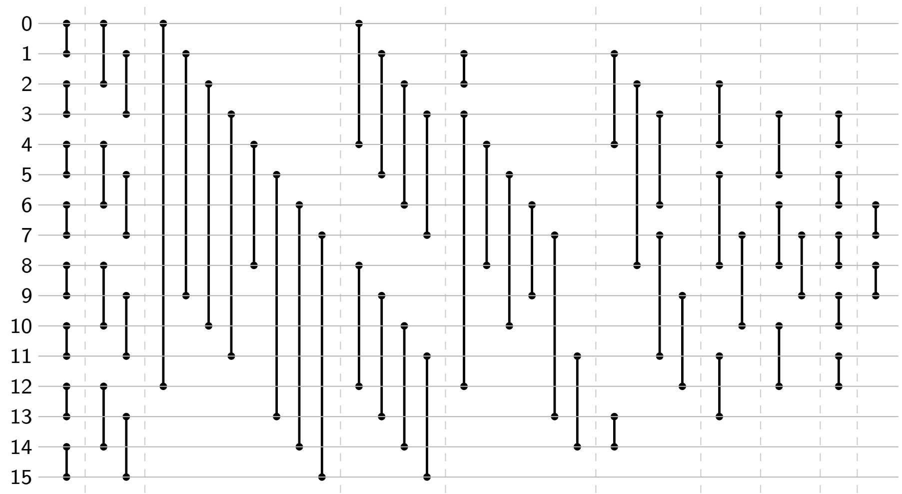
not reversal-symmetric; layers 1, 2, 5, 6, 7, 8, 9, 10 individually symmetric.




```
L1   (0,1) (2,3) (4,5) (6,7) (8,9) (10,11) (12,13) (14,15)
L2   (0,2) (1,3) (4,6) (5,7) (8,10) (9,11) (12,14) (13,15)
L3   (0,12) (1,9) (2,10) (3,7) (4,8) (5,13) (6,14) (11,15)
L4   (0,4) (1,5) (2,6) (3,11) (7,15) (8,12) (9,13) (10,14)
L5   (1,2) (3,12) (4,8) (5,10) (6,9) (7,11) (13,14)
L6   (1,4) (2,8) (3,6) (7,13) (9,12) (11,14)
L7   (2,4) (5,8) (7,10) (11,13)
L8   (3,5) (6,8) (7,9) (10,12)
L9   (3,4) (5,6) (7,8) (9,10) (11,12)
L10  (6,7) (8,9)
```

Network 16 $L_3 = (\text{fold},\ s_4,\ s_2)$

not reversal-symmetric; layers 1, 2, 4, 7, 8, 9, 10 individually symmetric.



```
L1   (0,1) (2,3) (4,5) (6,7) (8,9) (10,11) (12,13) (14,15)
L2   (0,2) (1,3) (4,6) (5,7) (8,10) (9,11) (12,14) (13,15)
L3   (0,12) (1,9) (2,10) (3,11) (4,8) (5,13) (6,14) (7,15)
L4   (0,4) (1,5) (2,6) (3,7) (8,12) (9,13) (10,14) (11,15)
L5   (1,2) (3,12) (4,8) (5,10) (6,9) (7,13) (11,14)
L6   (1,4) (2,8) (3,6) (7,11) (9,12) (13,14)
L7   (2,4) (5,8) (7,10) (11,13)
L8   (3,5) (6,8) (7,9) (10,12)
L9   (3,4) (5,6) (7,8) (9,10) (11,12)
L10  (6,7) (8,9)
```

Summary

network	L_3 program	reversal-symmetric	symmetric layers
01	$(s_1,\ s_2,\ s_1)$	fully	10/10
07	$(s_2,\ s_2,\ s_2)$	fully	10/10
10	$(s_2,\ s_4,\ s_2)$	fully	10/10
14	$(\mathrm{fold},\ s_2,\ \mathrm{fold})$	fully	10/10
02	$(s_1,\ s_2,\ s_2)$	as a set	6/10
04	$(s_1,\ s_4,\ s_2)$	as a set	6/10
06	$(s_2,\ s_2,\ s_1)$	as a set	6/10
09	$(s_2,\ s_4,\ s_1)$	as a set	6/10
03	$(s_1,\ s_2,\ \mathrm{fold})$	no	6/10
05	$(s_1,\ s_4,\ \mathrm{fold})$	no	8/10
08	$(s_2,\ s_2,\ \mathrm{fold})$	no	9/10
11	$(s_2,\ s_4,\ \mathrm{fold})$	no	7/10
12	$(\mathrm{fold},\ s_2,\ s_1)$	no	6/10
13	$(\mathrm{fold},\ s_2,\ s_2)$	no	9/10
15	$(\mathrm{fold},\ s_4,\ s_1)$	no	8/10
16	$(\mathrm{fold},\ s_4,\ s_2)$	no	7/10

All sixteen are distinct comparator sequences, all of size 60 and depth 10, all verified exhaustively on the 2^{16} binary inputs.

This is an incomplete survey. These sixteen are what a search under heuristic constraints found in around ten minutes, by interpreted Python running on a single core of an AMD Ryzen 5 5500U at 4GHz.